INSEAD

The Business School for the World®

Market States and Entry/Exit in Perfect Competition

Ekin Ilseven

Thoughts developed in the class Microeconomics A, P1 2017

November 23, 2017

1 Introduction

In this paper, we construct a way of analyzing behavior of firms in a given market. For this purpose, we first look at entry and exit conditions into a perfectly competitive market. We assume that all the firms have the same cost structure $C(Q)$, where Q denotes quantity of the product which can be continuous or discrete, and all of them makes the same decisions instantaneously, meaning that any change to the cost structure will be identical to all firms. This means that the firms do not have any competitive advantage over any other one; hence we assume perfect competition in the market. As the main purpose is investigating entry and exit, we will analyze only firm behavior and take demand (described by demand curve $D(P)$, where P denotes price given in some currency) and the cost structure of the firms as exogenous variables.

2 Market Price

We determine the market price, P_M through the market supply/demand curves, denoted as $S(P)$ and $D(P)$ respectively:

$$D(P_M) = S(P_M), \tag{1}$$

and the market supply curve $S(P)$ is obtained through the sum of individual supply curves $s(P)$:

$$D(P_M) = Ns(P_M), \tag{2}$$

where N denotes the number of firms in the market. It is known that unless the curves intersect, there will be no trade, hence the market will not work. So we will look only at the markets where the intersection exists and the point of intersection is taken as an equilibrium point. The solutions are limited by the condition that N is a natural number, which means that there are countable many prices that can satisfy this equation. We will define the set of prices as following:

$$\mathbb{P} = \left\{ \mathrm{p} \in \mathbb{R} | \frac{\mathrm{D(p)}}{\mathrm{s(p)}} = \mathrm{N} \in \mathbb{N} \right\} \tag{3}$$

In a free market with perfect competition, the realized market price will be the minimum of this set, $P_M = \min(\mathbb{P})$, because as long as a lower price can exist, someone will enter the market to drive the price down by offering more of the product. An important observation has to be pointed out at this point:

Observation: *Everytime a firm enters or exists the market, the price changes in discrete jumps.*

This observation makes sense as entry shifts the supply curve (the increase in offer of product) discretely. For later purposes, the difference of a firm entering the market is presented here:

$$N_2 - N_1 = \frac{D(p_2)}{s(p_2)} - \frac{D(p_1)}{s(p_1)} = \frac{D(p_2)s(p_1) - D(p_1)s(p_2)}{s(p_2)s(p_1)} = 1. \tag{4}$$

Once the demand and supply functions are given explicitly, we can calculate p_2 from p_1 in an iterative way and vice versa. As we treat demand as an exogenous factor, we take a closer look at supply in the next section.

3 Supply Curve

A price-taking firm (price-taking due to perfect competition) wants to maximize its profits given a price to the product and profits are calculated through the difference of revenue and costs. This means that a firm wants to solve the following maximization problem:

$$\max\left(PQ - C(Q)\right), \tag{5}$$

where P is the price of the product to be determined by the firm and $C(Q)$ is taken as exogenous. This condition corresponds to the Legendre transformation of a differentiable function C, thus the price that maximizes the quantity depending on price is the solution of $P = dC/dQ = MC(Q)$, where $MC(Q)$ is the marginal cost of the firm. A firm will produce the quantity that maximizes its profit, which in other words means that the firm will supply the product to the market at the quantity that maximizes its profits, namely at $MC^{-1}(P)$. To obtain the supply curve, we simply invert this relation $s(P) = MC^{-1}(P)$. While this calculation gives us a supply curve, the lower boundary of the supply curve has to be determined as well. If the price of the product falls below the average cost of the product $AC(Q)$, the firm will not make *any* profit. So the supply curve exists only for the interval where $MC(Q) \geq AC(Q)$.

4 Perfectly Competitive Free Market Solution

Using the results derived in the previous sections, we can present now the accurate set of prices, which depend on firm's cost structure:

$$\mathbb{P} = \left\{ \mathrm{p} \in \mathbb{R} | \frac{\mathrm{D(p)}}{\mathrm{MC^{-1}(p)}} = \mathrm{N} \in \mathbb{N} \wedge \mathrm{AC(MC^{-1}(p))} \leq \mathrm{p} \right\}. \tag{6}$$

From this set we can derive what will be the market price P_M and how many firms there will be, N. While the relation between P_M and N requires more investigation, a canonical case would be that $P_M = \min(\mathbb{P})$ is equivalent to picking the maximum N.

4.1 Example 1.1

This case we will examine with a simple example. Let's assume that $D(p) = d - p$, where d is the price which would be paid for the very first product and p the price. As the cost structure we choose

$$C(Q) = \frac{Q^3}{3} + F, \quad MC(Q) = Q^2, \quad AC(Q) = \frac{Q^2}{3} + \frac{F}{Q}, \tag{7}$$

where F denotes the fixed cost. In a market, where there are no fixed costs, firms will enter without a bound. Let's demonstrate this by setting $F = 0$. First we look at the condition $AC(MC^{-1}(p)) \leq p$. Using the cost structure, we see that we have on the left hand side $p/3$ and on the right hand side we have p. This means that this condition is fulfilled for all $p > 0$. As next we look at the solutions of the equation $d - p - N\sqrt{p} = 0$, which is the first condition in the definition of $\mathbb{P}$. A quick calculation shows that

$$p_N = \frac{N^2}{4}\left(-1 + \sqrt{1 + \frac{4d}{N^2}}\right)^2, \tag{8}$$

hence the solution always exists and the firms always make profit which is exactly 2/3 of this expression. In this case, N will tend to infinity and P_M will tend to zero. So to have a more interesting system we have to include the fixed costs in this model.

4.2 Example 1.2

In this example we look at non-zero fixed costs and determine the maximum number of firms in a market, the price of such a system and how much profit these firms make. For this we first illustrate with some fixed numbers for the constants d and F. For non zero F the condition $AC(MC^{-1}(p)) \leq p$ becomes

$$p_N \geq \left(\frac{3F}{2}\right)^{\frac{2}{3}}. \tag{9}$$

For example choosing, $d = 100$ and $F = 100$, we find with the given constraint that

$$\mathbb{P} = \{29.45, 32.05, 34.96, \dots\}, \tag{10}$$

where the set is presented in an ascending order with 13 elements in total. This means that there will 13 firms in this market and the product will be sold for 29.45 unit currency. If a 14th firm enters the market, he would drive the prices down to 27.11 and following we will see the profits made for both cases. We calculate the average profit as $p - AC(MC^{-1}(p))$:

$$N = 13, p_{13} = 29.45 \implies 29.45 - AC(MC^{-1}(29.45)) = 29.45 - \frac{29.45}{3} - \frac{100}{\sqrt{29.45}} = 1.21, \tag{11}$$

$$N = 14, p_{14} = 27.11 \implies 27.11 - AC(MC^{-1}(27.11)) = 27.11 - \frac{27.11}{3} - \frac{100}{\sqrt{27.11}} = -1.13. \tag{12}$$

We see clearly that an extra firm in the market leads to negative profits for all the firms. In this case another observation can be made:

Observation: *With fixed demand and cost structure, the firms will make profit due to the discreteness of the possible prices.*

This observation leads to two mechanism that should be observed next. How does the number of firms and their profits change when d or F change? For this first, we visualize this analysis. The curves are presented in Fig. 1. The purple and yellow thick curves represent the market supply for 13 and 14 firms in the market respectively. The blue thick curve is the demand curve and their intersection determines the price. The red thick line shows the limiting price: to the left of the red curve the firms would lose money, while to the right the would make profit. This certain price limits the number of firms entering the market, as well as leads to non zero profit depending on its value. For completeness, we presented also the market supply curves of markets with different numbers of firms in thin lines. Now we take a closer look at how the fixed cost will affect the profits and the number of firms in the market.

4.3 Fixed cost change

Changing fixed cost means shifting the red curve to the right or left. In Fig. 1, it can be seen that if the red curve shifts towards left enough, then a 14th firm will enter the market as the market price with 14 firms will switch to the right side of the red curve. While the effect of fixed costs is clear from the graph, its effect on the profits is more difficult to infer. In Fig. 2 we present the profit for the above given system, where demand curve is kept constant but the fixed cost drops from 574 (a fixed cost where nobody would make profit) to 100, where the market has 13 firms. The behavior of the price can be summarized with the following observations.

Observations: *i) Fixed cost is inverse proportional to profit. ii) The frequency of the entries to the market increases with falling fixed costs. iii) With every new firm entering the market, the maximum amount of profit that can be reached through adjustment of fixed cost falls.*

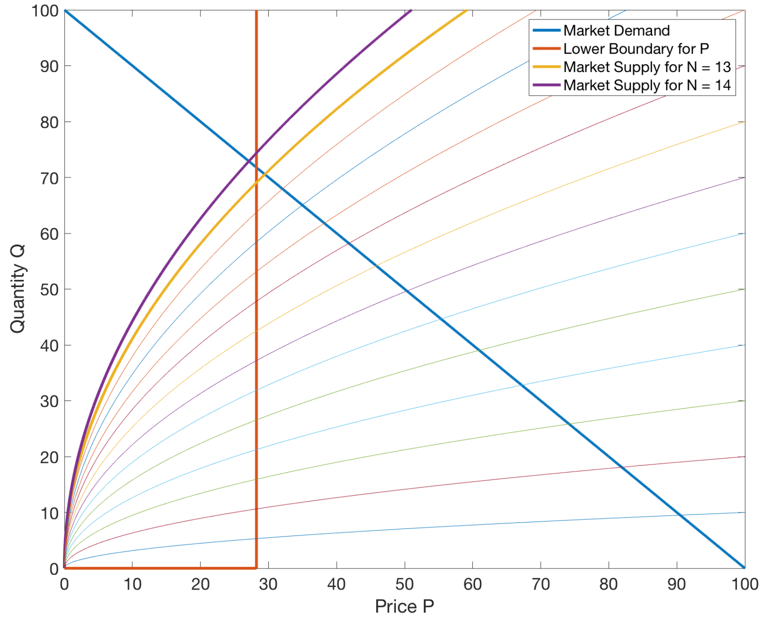

Figure 1: The curves can be seen

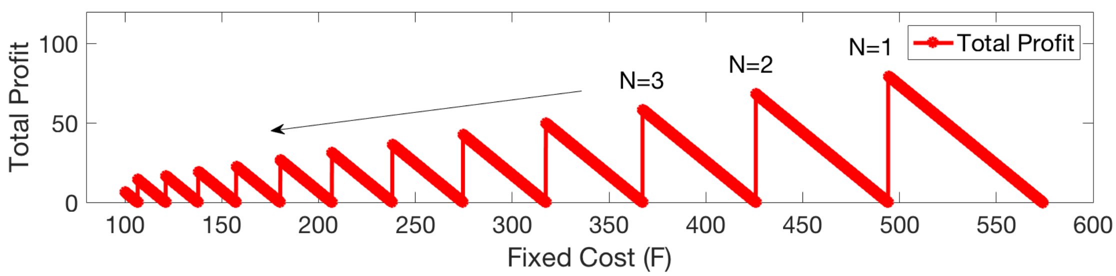

Figure 2: The curves can be seen

4.4 Demand change

Now we keep the fixed cost fixed at $F = 100$ and change the demand d from 0 to 100. Fig. 3 presents the results. We can observe the following in the graph.

Observations: *i) Demand is proportional to profit. ii) The frequency of the entries to the market does not change. iii) With every new firm entering the market, the maximum amount of profit that can be reached through increase in demand falls.*

Now we continue with explaining the observations.

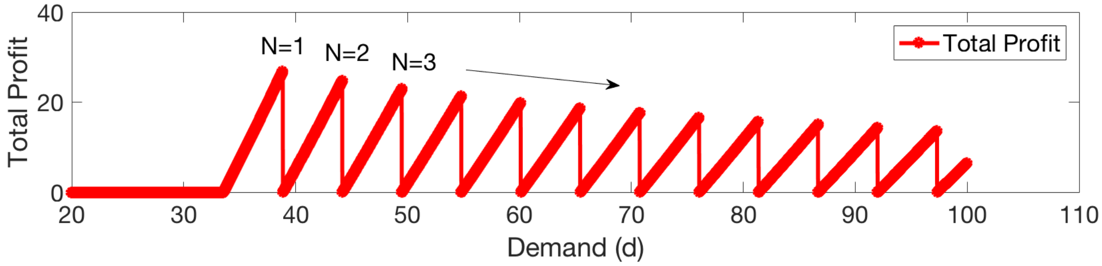

Figure 3: The curves can be seen

5 Analysis of Effects

5.1 Fixed cost change

5.1.1 Observation *i*

Fixed cost is inverse proportional to profit, which can be directly seen in the calculation of profit:

$$\text{Profit} = P_M MC^{-1}(P_M) - C(MC^{-1}(P_M)) = \frac{2P_M^{\frac{3}{2}}}{3} - F. \tag{13}$$

The market price P_M is fixed because the demand is fixed. P_M being constant means that this equation is in the form $y(x) = a - bx$, where variable is F. The minus sign in front of F explains why it is falling and the slope corresponds to 1; hence the slope is increasing with falling market prices, meaning the more firms there are in the market, quicker the profit will fall with increasing fixed costs.

5.1.2 Observation *ii* and *iii*

We explain these two observations together. Above we have seen that the slope increase with more firms entering the market, however the increase in the frequency limits the maximum profit that can be achieved with the increase in the slope. So, we first explain how the frequency changes and then show that the maximum falls as well. To explain the frequency we will use the distance between two zero profit points. At the points of zero profit, using Eq. 9, we see that the following condition must be fulfilled:

$$p = \left(\frac{3F}{2}\right)^{\frac{2}{3}} \implies F = \frac{2p^{\frac{3}{2}}}{3} \tag{14}$$

where p denotes the price. However as the market price depends on the number of firms in the market, p depends on N, hence F as well. Using this relation, we can calculate the distance between two zero profit points:

$$\Delta F = F(N) - F(N-1) = \frac{2}{3}\left(P_M(N)^{\frac{3}{2}} - P_M(N-1)^{\frac{3}{2}}\right). \tag{15}$$

Using Eq. 8, we can obtain an explicit form for the decrease in the distance. Here we will argue that the distance is always bounded and converges to the limit 0 as $N \to \infty$, hence the frequency $(1/\Delta F)$ has to increase monotonously and approach infinity with increasing N. Now the maximum of the profit can be calculated. For a constant slope between two zero profit points, the maximum profit for a market with $N-1$ firms is given by

$$\text{Profit}_{\max} = \Delta F = \frac{2}{3}\left(P_M(N)^{\frac{3}{2}} - P_M(N-1)^{\frac{3}{2}}\right). \tag{16}$$

With a similar argument as above, we can say that for big N, $P_M(N-1)^{\frac{1}{2}} \to P_M(N)^{\frac{1}{2}}$ and the maximum profit goes monotonously to zero.

5.2 Demand change

5.2.1 Observation *i*

Inserting Eq. 8 in Eq. 13, we obtain

$$\text{Profit} = P_M MC^{-1}(P_M) - C(MC^{-1}(P_M)) = \frac{N^3}{12}\left(-1 + \sqrt{1 + \frac{4d}{N^2}}\right)^3 - F. \tag{17}$$

For fixed N and F, the first term is proportional to the demand; hence the profit is proportional to demand.

5.2.2 Observation *ii*

We observe that the frequency stays constant. The explanation for this can be easily seen in the Fig. 1. All supply curves are multiples of a single supply curve (supply curve of a market with one firm), which means that each curve crosses the red thick line in equidistant points. So calculating the value of supply curve for one firm at the price shown by the red thick line gives us the distance between each curve crossing the same red thick line. For $F = 100$ the red line lies at 28.23, and calculating $MC^{-1}(28.23)$ gives 5.31, which is the constant distance between all the intersection of supply curves on the red line. We can generalize this distance further as:

$$\Delta d = MC^{-1}\left(\left(\frac{3F}{2}\right)^{\frac{2}{3}}\right) = \left(\frac{3F}{2}\right)^{\frac{1}{3}}. \tag{18}$$

Every time the demand curve crosses these equidistant points lying on the red line a new firm will enter the market.

5.2.3 Observation *iii*

We see in Eq. 17 that the profit has a relatively complicated form and not a constant slope as previously. Nevertheless we can know that the profit goes back to zero with the entry of every new firm and the points where the profit would be zero if there was an extra firm in the market correspond to the maximum profit that could be achieved if there is not. So we can calculate the difference as following:

$$\Delta\text{Profit}_{\max} = \text{Profit}_N((N+1)\Delta d) - \text{Profit}_{N-1}(N\Delta d). \tag{19}$$

Although a rigorous proof can be certainly given, we use the arguments given in the previous section and use the fact that the difference is bounded and reaches the limit 0 for $N \to 0$, hence the maximum profit should fall monotonously.

5.3 Summary

For the given functional forms of cost structure and demand, we map out our findings for the profit in two dimensions d and F in Figure 4. The yellow color denotes higher profits and dark blue is 0 profit. Our analysis was along the left and top edges of this plot, where the profit grows and falls to zero periodically with decreasing fixed costs and increasing demand. An interesting observation is that staying on the maximum profit frontier on any non-zero profit region and increasing fixed costs according to the demand increases the profits (see bright yellow color on the top edge vs. the green-blue colors on the left edge).

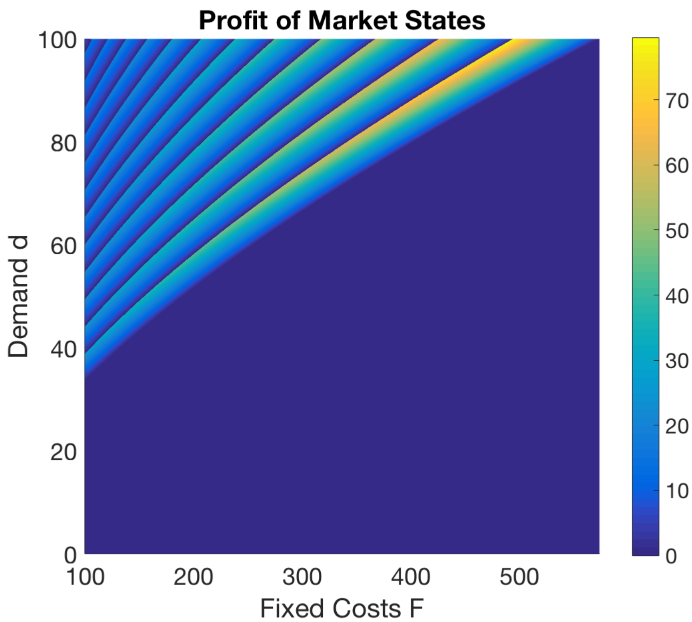

Figure 4: Profits of Markets with N firms

6 Market States and Firm Dynamics

6.1 Market States, Profitability and Transitions

The set $\mathbb{P}$ defines pairs of firm numbers and price ranges $(N, p_N(d, F))$ at which these firms would operate and these pairs are the transactions in reality; the potential behaviors of the firms do not affect this structure. Hence, we can take these pairs as states of market which depend on some characteristics of supply and demand. In Figure 4, the first non-zero profit region to the bottom right is a one firm market $(N = 1)$ and the two dimensional region defines a range of prices $p_1(d, F)$ such that $(d, F) \in \text{Domain}(\text{Market}_{N=1})$. We will call each of these regions the domain of a market state and a change from one state to another can be achieved by some firms entering or exiting. For example, if a firm enters the market, the market transits from state $(N, p_N(d, F))$ to $(N+1, p_{N+1}(d, F))$. Furthermore, each state has a profitability. Those which are very profitable are highly unlikely to be realized, whereas those which are not profitable neither. In the ideal case, the market will equilibrate at the minimum positive profitability value.

The zeros of the profit function separate one state from the other. These lines are denoted as $p^*_N(d, F)$. As already demonstrated, the zero profits of the N firm market state are maximum profits of $N + 1$ firm market state. Hence, we define regions S_N which correspond to the states of the market where the firms can make profit. In the previous sections, we calculated the distance between two states along constant demand or constant fixed cost lines. For example, the zero profit lines for such a market can be calculated by setting Equation 17 equal zero and determining the pairs of d and F which depend on N. The transition of the market from one state to another is described by transition probabilities. Assuming that only one firm can enter or exit the market, we can say that for the S_N the transition can happen only from or to S_{N+1} or S_{N-1}. We denote this as following:

$$S_{N+1} \rightleftharpoons S_N \rightleftharpoons S_{N-1}$$

The transition probability from one state to the other depends on the profit made. There are four transition probabilities from and to a randomly chosen state S_N: The first two correspond to entries from outside (1: $S_{N-1} \to S_N$) and (2: $S_N \to S_{N+1}$). The next two are exits from inside (3: $S_N \to S_{N-1}$) and (4: $S_{N+1} \to S_N$). For a perfectly competitive free markets, we would see that transitions 1 and 2 would happen with certainty if there is a potential profit to be realized, whereas the transitions 3 and 4 would happen when the firms start to make losses. However, we expect these transition probabilities to differ from 1 or 0, for imperfect competition, especially when the profits get closer to the borders of the state. These four transitions correspond to premature actions. Transition 1 or 2 might happen, if a firm can afford making losses but wants to secure market share or drive prices down for a strategic reason. Similarly transition 3 or 4 might happen if a firm wants to relocate its resources in another market and does not find this market profitable enough for a reason. While a closer look is necessary to understand the nature of these probabilities, we will use some abstract forms to facilitate further analysis of the dynamics. On the other hand, there are also stickiness of firms which reduce the effect of the premature actions. The stickiness corresponding to transitions 1 and 2 would be the reluctance of any firm outside to enter the market or the inability of entry of the firms; hence even if the market is profitable for entry, the market does not make a jump. For transitions 3 and 4 we can say that a firm sticks to the market for a reason such as barriers of exit, emotional attachments, strategic reasons. To summarize, we see four forces at work in this model: 1) Premature entry, 2) Premature exit, 3) Sticky Non-Entry and 4) Sticky Non-Exit. While the forces 1 and 4 work in favor of the state at hand, 2 and 3 work against it.

6.2 Expected Profit and Market State

These transitions lead to a dynamic state of the firms. Here, we take the transitions as probabilistic processes and demonstrate its effects on the expected profits over time and the expected state of the market for a given pair of (d, F). We keep fixed cost $F = 100$ constant and vary d as the dynamics are going to be very similar and this will facilitate the analysis. Using Δd from previous analysis and solving for the first zero of the profit along the demand line on the left edge, we see that jumps happen at the points

$$d_N^* = \left(\frac{3F}{2}\right)^{\frac{2}{3}} + (N-1)\left(\frac{3F}{2}\right)^{\frac{1}{3}}. \tag{20}$$

We choose the unit vector representation for the states, i.e. $S_N = (0, \ldots, 0, 1, 0, \ldots, 0)$ where 1 is at the Nth entry, and will look at the premature transition effects in the market, which we will represent with a transition probability matrix $T_{NM}(d)$, such that $S^{t+\delta} = T(d)S^t$. δ denotes a time step, which is associated with the application of matrix here. Now we construct $T(d)$ for the given rate diagram above. First we look at the limiting behavior: All firms can enter and exit instantaneously and they follow the profitability condition perfectly.

$$T = \begin{bmatrix} T_{N+1,N+1} & T_{N,N+1} & T_{N-1,N+1} \\ T_{N+1,N} & T_{N,N} & T_{N-1,N} \\ T_{N+1,N-1} & T_{N,N-1} & T_{N-1,N-1} \end{bmatrix}$$

1. $S_N \to S_{N+1}$: The transition happens only for $d > d_{N+1}^*$.
2. $S_{N-1} \to S_N$: The transition happens only for $d > d_N^*$.
3. $S_{N+1} \to S_N$: The transition happens only for $d < d_{N+1}^*$.
4. $S_N \to S_{N-1}$: The transition happens only for $d < d_N^*$.

Furthermore, we denote the cases where firms do not take action:

1. $S_{N+1} \to S_{N+1}$: The transition happens only for $d^*_{N+1} > d > d^*_{N+2}$.
2. $S_N \to S_N$: The transition happens only for $d^*_N > d > d^*_{N+1}$.
3. $S_{N-1} \to S_{N-1}$: The transition happens only for $d^*_{N-1} < d < d^*_N$.

Using the heaviside function $\theta(x - x') = 1, \ \ x > x'$, we find the explicit form of T:

$$T(d) = \begin{bmatrix} \theta(d^*_{N+2} - d)(1 - \theta(d^*_{N+1} - d)) & 1 - \theta(d^*_{N+1} - d) & 0 \\ \theta(d^*_{N+1} - d) & \theta(d^*_{N+1} - d)(1 - \theta(d^*_N - d)) & 1 - \theta(d^*_N - d) \\ 0 & \theta(d^*_N - d) & \theta(d^*_N - d)(1 - \theta(d^*_{N-1} - d)) \end{bmatrix}$$

The expectation value of the number of firms and profits can be calculated through

$$E[N; d, F] = \sum_N N \cdot S_N(d, F), \quad E[P] = \sum_N P(N; d, F) \cdot S_N(d, F). \tag{21}$$

The Figure 3 shows the expectation value of profits and the number of firms (as labeled) as the demand d grows slowly over time. As next we will extend the transition matrix canonically. The heaviside function can be taken as the zero temperature limit of Fermi-Dirac distribution, which is given as

$$\lim_{\tau \to 0} f(\tau, E, E_F) = \lim_{\tau \to 0} \frac{1}{e^{\frac{E - E_F}{\tau}} + 1} = \theta(E_F - E). \tag{22}$$

We will use the parameter τ as a degree of premature action taken by one firm in or out of the market (respectively exit and entry actions). $\tau = 0$ means there will not be any premature action and corresponds to the matrix given above. The matrix in its general form is given as:

$$T(d) = \begin{bmatrix} f(\tau, d, d^*_{N+2})(1 - f(\tau, d, d^*_{N+1})) & (1 - f(\tau, d, d^*_{N+1})) & 0 \\ f(\tau, d, d^*_{N+1}) & f(\tau, d, d^*_{N+1})(1 - f(\tau, d, d^*_N)) & (1 - f(\tau, d, d^*_N)) \\ 0 & f(\tau, d, d^*_N) & f(\tau, d, d^*_N)(1 - f(\tau, d, d^*_{N-1})) \end{bmatrix}$$

In Figures 5 and 6, we present the expectation value of state number and expected profits for $\tau = (0.01, 0.1, 0.5, 1)$, where we used 5000 timesteps ($\delta = 1 \to 5000$) in which d goes from 35 to 100. The premature action near the transition points smears the maximum and minimum of the profit function; the maximum profit that can be made diminishes while the zero profit points vanish and all profit becomes positive for all demand.

7 Conclusion and Outlook

We have investigated the entry and exit conditions in a free market with perfect competition first in a general form, then concretely with a fixed structure given to demand and cost of the firms. The main outcome of this work shows that the assumptions of identical firms and instantaneous changes in the economic system lead to a discrete set of realizable prices and this reflects itself in countably many sets of zero profit followed by non-zero unavoidable profit between these points. We have analyzed the effects of demand and fixed costs on the magnitude of the profit and pointed out some characteristics of the changes in profit. To investigate the firms behaviors, we used this discrete structure as an underlying state structure and took the firms as populations moving from one state to another. We identified four possible strategies that could affect the profit and number patterns and looked at its effects on number of firms in the market and the expected profit from the market.

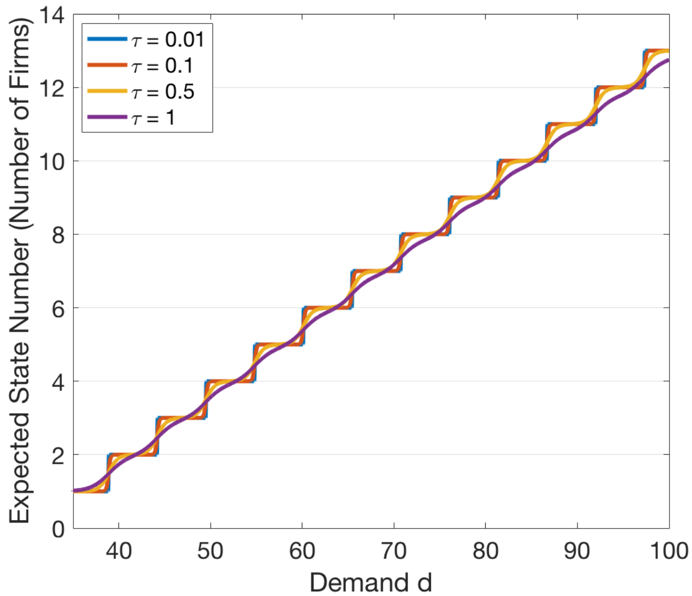

Figure 5: Expected number of firms in the market with respect to demand d

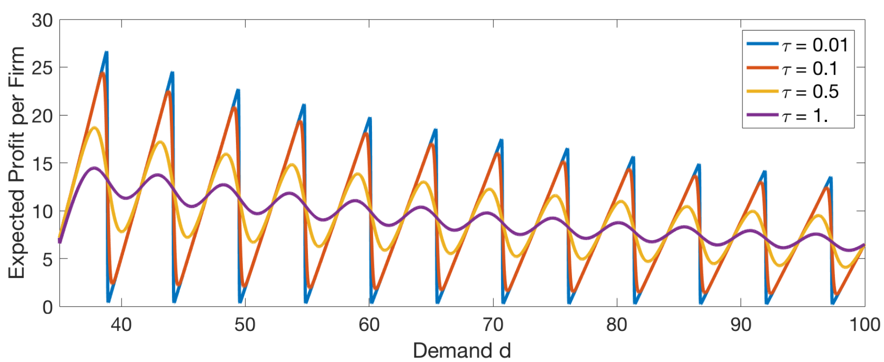

Figure 6: Expected profits of firms with respect to demand d

The main purpose of this work is to underline the difference between a market structure and the firms behaviors on this structure. The market structure is what is measured in reality and the firms experience the profit which corresponds to the market state at hand. On the other hand, when the firms make a decision, they are not able to predict exactly what the market state will be, hence they will base their decisions based on the expected market state. By introducing this separation, we aim at separating equilibrium calculations in microeconomics from firms' decision making processes and their behaviors, achieving a degree of measurability of firm dynamics.

A shortcoming of this model is that the firms strategies change the expected the profit, which should in return change the behavior of the firms. However in this model we looked at the case where the firms do not know the expected profit or number of firms in the market. While game theoretical considerations may be successful to explain these adjustments, a more orthodox self-feedback mechanism (such as an analogy from Physics, specifically local density approximation in density functional theory) could be incorporated in this work. Furthermore, the transition probabilities require more elaboration: i) There are only nearest-neighbor transitions, whereas in reality two firms might want to exit or enter at the same time as well. This should affect the expected market price in some way. ii) The transition probabilities might be characteristic to the market or might be dependent on the firms in the market (e.g. multi-market firms will have different behavior compared to those performing only in the market at hand). The transition probabilities need more attention. While the structure we chose for demand and cost simplified the calculations, they were helpful in the primary analysis of the discrete effects. Further work should also choose different structures and, if possible, generalize the work in any structure. A possible next step would be looking at the effects the elasticities of supply and demand on the trends that were observed in these examples. The elasticity of demand may affect the frequency of entries due to a change in fixed cost or the maximum profits that can be achieved.

8 Acknowledgements

Thanks to Natalie Burford for her inspiring question in the Microeconomics class, *"What if the firms expanded a little bit?"*, posed at the remark of our professor Timothy Van Zandt that the number of firms entering a free market was assumed to be a real number and should be rounded down. Many thanks to Andres Alban for the inspiring follow-up discussions.